

Nuclear moments for the neutrinoless double beta decay II

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Abstract

The recently developed formalism for the evaluation of nuclear form factors in neutrinoless double beta decay is applied to ^{48}Ca , ^{76}Ge , ^{82}Se , ^{100}Mo , ^{128}Te and ^{130}Te nuclei. Explicit analytical expressions that follows from this theoretical development, in the single mode model for the decay of ^{48}Ca , have been worked out. They are useful both for testing the full numerical calculations, and for analytically checking the consistency with other formalisms. Large configuration space calculations are compared with previous studies, where alternative formulations were used. Yet, besides using the G-matrix as residual interaction, we here use a simple δ -force. Attention is paid to the connected effects of the short range nuclear correlations and the finite nucleon size. Constraints on lepton number violating terms in the weak Hamiltonian (effective neutrino Majorana mass and effective right-handed current coupling strengths) are deduced.

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1 Introduction

During the last years we have developed a new formulation for the neutrinoless double beta ($\beta\beta_{0\nu}$) decay, based on the Fourier-Bessel multipole expansion of the hadronic current, and on the angular momentum recoupling. First, we did it for the mass term within the single mode model (SMM) [1]. Later on, full QRPA calculations were done for this term [2]. More recently, the same procedure has been applied to the evaluation of the so called "recoil term" in the charged Majoron emission [3]. Finally, the complete formalism, including the right-handed ($V + A$) hadronic current, was presented [4]. The physical substratum in this development is the same as in the previous works on the same issue [5, 6, 7, 8, 9], namely, the same weak Hamiltonian was used. Thus, one cannot expect to get sensibly different results for the corresponding observables. Yet, we have succeeded in expressing all nuclear $\beta\beta_{0\nu}$ moments in terms of the matrix elements of *only three* well-known one-body spherical tensor operators:

$$\begin{aligned} Y_{\lambda JM}^\kappa(k) &= \sum_n \tau_n^+ r_n^\kappa j_\lambda(kr_n) Y_{JM}(\hat{\mathbf{r}}_n), \\ S_{\lambda L JM}^\kappa(k) &= \sum_n \tau_n^+ r_n^\kappa j_\lambda(kr_n) [\boldsymbol{\sigma}_n \otimes Y_L(\hat{\mathbf{r}}_n)]_{JM}, \\ P_{L JM}(k) &= \sum_n \tau_n^+ j_L(kr_n) [\mathbf{p}_n \otimes Y_L(\hat{\mathbf{r}}_n)]_{JM}, \end{aligned} \tag{1}$$

which have been around in nuclear physics for more than 40 years [10, 11]. This makes our formulation to be specially suitable for the nuclear structure calculations, and more simple than other formulations [5, 6, 7, 8, 9].

In fact, the Fourier-Bessel multipole expansion has also been used by Vergados [8] and by Suhonen, Khadkikar and Faessler [9], as the starting point. However, the final outcomes for the nuclear matrix elements in these two theoretical developments are quite dissimilar, not only to our formulas, but also to each other. They are also different to the formulas derived by Haxton and Stephenson [5] and by Doi, Kotani and Takasugi [6] within the closure approximation. As a consequence, the alternative formalisms cannot be confronted analytically, and the only way to test the consistency among them is by way of numerical procedures.

As far as we know, numerical calculations, using the formalism from ref. [4], have so far been performed only for the neutrino mass term [2, 12]. Thus, to complete our study on the $\beta\beta_{0\nu}$ matrix elements, in this paper we carry out calculations for several $\beta\beta$ decaying nuclei that are attractive from the experimental point of view (^{48}Ca , ^{76}Ge , ^{82}Se , ^{100}Mo , ^{128}Te and

^{130}Te). A comparison with similar studies is also made.

As most of the previous studies were performed in the framework of the QRPA model [12, 13, 14, 15, 16], we will use here mostly the same nuclear structure approach. Only the $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$ -decay will be discussed in a simple shell model, in order to compare our formalism with that of Vergados [8]. Also, for the sake of comparison, throughout this work the bare axial vector coupling constant $g_A = 1.254$ will be used, although the effective value $g_A^{eff} = 1$ is preferable in nuclear physics [17, 18].

The outline of this paper is as follows:

In Section 2 we summarize the main results for the new formalism developed recently [4].

In Section 3 the decay of ^{48}Ca is discussed within the SMM and a comparison is made with the shell model calculation performed by Pantis and Vergados [19]. Here we also derive the analytical expressions for nuclear moments, which we later use to test the numerical results. The competition between the effects of the finite nucleon size (FNS) and the two-nucleon short range correlations (SRC) is discussed as well.

Section 4 deals with full QRPA calculations for ^{48}Ca , ^{76}Ge , ^{82}Se , ^{100}Mo , ^{128}Te and ^{130}Te . We compare them with two previous QRPA evaluations, namely with: 1) the results obtained by Muto, Bender and Klapdor (MBK) [14], where the formalism of Doi, Kotani and Takasugi [6] has been utilized, and 2) the calculation performed by Pantis, Šimkovic, Vergados and Faessler (PSVF) [15] in the framework of the formalism developed by Vergados [8]. In this section we also show the limits on the $\beta\beta_{0\nu}$ coupling constants that we deduce from the most recent experimental data [20, 21, 22, 23, 24].

Concluding remarks are pointed out in Section 5.

2 Neutrinoless double beta decay formalism

The $\beta\beta_{0\nu}$ half-life is expressed in the standard form [7]:

$$\begin{aligned} [T_{0\nu}(0^+ \rightarrow 0^+)]^{-1} &= \langle m_\nu \rangle^2 C_{mm} + \langle \lambda \rangle^2 C_{\lambda\lambda} + \langle \eta \rangle^2 C_{\eta\eta} \\ &+ \langle m_\nu \rangle \langle \lambda \rangle C_{m\lambda} + \langle m_\nu \rangle \langle \eta \rangle C_{m\eta} + \langle \lambda \rangle \langle \eta \rangle C_{\lambda\eta}, \end{aligned} \quad (2)$$

where $\langle m_\nu \rangle$ is the effective neutrino mass and $\langle \lambda \rangle$ and $\langle \eta \rangle$ are the effective coupling constants of the $(V + A)$ hadronic currents. The coefficients

$$\begin{aligned} C_{mm} &= (M_F - M_{GT})^2 \mathcal{G}_1, \\ C_{\lambda\lambda} &= M_{2-}^2 \mathcal{G}_2 + \frac{1}{9} M_{1+}^2 \mathcal{G}_4 - \frac{2}{9} M_{2-} M_{1+} \mathcal{G}_3, \\ C_{\eta\eta} &= M_{2+}^2 \mathcal{G}_2 + \frac{1}{9} M_{1-}^2 \mathcal{G}_4 - \frac{2}{9} M_{2+} M_{1-} \mathcal{G}_3 + M_R^2 \mathcal{G}_9 + M_R M_P \mathcal{G}_7 + M_P^2 \mathcal{G}_8, \\ C_{m\lambda} &= (M_F - M_{GT}) [M_{2-} \mathcal{G}_3 - M_{1+} \mathcal{G}_4], \\ C_{m\eta} &= -(M_F - M_{GT}) [M_{2+} \mathcal{G}_3 - M_{1-} \mathcal{G}_4 + M_R \mathcal{G}_6 + M_P \mathcal{G}_5], \\ C_{\lambda\eta} &= -2M_{2-} M_{2+} \mathcal{G}_2 + \frac{2}{9} [M_{2-} M_{1-} + M_{2+} M_{1+}] \mathcal{G}_3 - \frac{2}{9} M_{1-} M_{1+} \mathcal{G}_4, \end{aligned} \quad (3)$$

contain the combinations of the matrix elements

$$\begin{aligned} M_{1\pm} &= M_{GT'} - 6M_T \pm 3M_{F'}, \\ M_{2\pm} &= M_{GT\omega} \pm M_{F\omega} - \frac{1}{9} M_{1\mp}. \end{aligned} \quad (4)$$

The kinematical factors \mathcal{G}_k are given by eq. (3.5.17) in ref. [6] and the nuclear matrix elements read [4]:

$$M_F = \left(\frac{g_V}{g_A} \right)^2 \sum_{J_\alpha^\pi p n p' n'} (-)^J \mathcal{W}_{J0J}(pn) \mathcal{W}_{J0J}(p'n') \mathcal{R}_{JJ}^{00}(pn p' n'; \omega_{J_\alpha^\pi}) \rho^{ph}(pn p' n'; J_\alpha^\pi), \quad (5)$$

$$M_{GT} = - \sum_{L J_\alpha^\pi p n p' n'} (-1)^L \mathcal{W}_{L1J}(pn) \mathcal{W}_{L1J}(p'n') \mathcal{R}_{LL}^{00}(pn p' n'; \omega_{J_\alpha^\pi}) \rho^{ph}(pn p' n'; J_\alpha^\pi), \quad (6)$$

$$\begin{aligned} M_{F'} &= -2 \left(\frac{g_V}{g_A} \right)^2 \sum_{L J_\alpha^\pi p n p' n'} i^{L+J+1} (J1|L)(J1|L) \mathcal{W}_{J0J}(pn) \mathcal{W}_{J0J}(p'n') \\ &\times \mathcal{R}_{JL}^{11}(pn p' n'; \omega_{J_\alpha^\pi}) \rho^{ph}(pn p' n'; J_\alpha^\pi), \end{aligned} \quad (7)$$

$$\begin{aligned}
M_{GT'} &= 2 \sum_{LL' J_\alpha^\pi pnp'n'} i^{L+L'+1} (L'1|L)(L'1|L) \mathcal{W}_{L'1J}(pn) \mathcal{W}_{L'1J}(p'n') \\
&\times \mathcal{R}_{L'L}^{11}(pnp'n'; \omega_{J_\alpha^\pi}) \rho^{ph}(pnp'n'; J_\alpha^\pi),
\end{aligned} \tag{8}$$

$$\begin{aligned}
M_R &= \frac{R}{2M_N} \frac{g_V}{g_A} \sum_{LL' J_\alpha^\pi pnp'n'} i^{L+L'} \mathcal{W}_{L1J}(pn) \rho^{ph}(pnp'n'; J_\alpha^\pi) \\
&\times \left\{ -f_W \left[\delta_{LL'} - (J1|L)(J1|L') \right] \mathcal{W}_{L'1J}(p'n') \mathcal{R}_{LL'}^{20}(pnp'n'; \omega_{J_\alpha^\pi}) \right. \\
&\left. - 2\sqrt{6} \hat{L} W(LJ11; 1L') (L1|L') \sum_{\kappa=\pm} \mathcal{W}_{L'J}^{(\kappa)}(p'n') \mathcal{R}_{LL'}^{1\kappa}(pnp'n'; \omega_{J_\alpha^\pi}) \right\},
\end{aligned} \tag{9}$$

$$\begin{aligned}
M_T &= 10 \sum_{LL' J' J_\alpha^\pi pnp'n'} i^{L+L'+1} \hat{L}^2 (1L|J')(1L|L') W(12LJ'; 1L') W(12JJ'; 1L') \\
&\times \mathcal{W}_{L'1J}(pn) \mathcal{W}_{J'1J}(p'n') \mathcal{R}_{L'L}^{11}(pnp'n'; \omega_{J_\alpha^\pi}) \rho^{ph}(pnp'n'; J_\alpha^\pi),
\end{aligned} \tag{10}$$

$$\begin{aligned}
M_P &= 2\sqrt{6} \frac{g_V}{g_A} \sum_{LJ_\alpha^\pi pnp'n'} i^{1-L-J} \hat{J} (J1|L)(J1|L) W(JL11; 1J) \\
&\times \mathcal{W}_{J0J}(pn) \mathcal{W}_{J1J}(p'n') \mathcal{R}_{JL}^{11}(pnp'n'; \omega_{J_\alpha^\pi}) \rho^{ph}(pnp'n'; J_\alpha^\pi).
\end{aligned} \tag{11}$$

Here R is the nuclear radius, M_N is the nucleon mass, the index α labels different intermediate states with the same spin J and parity π , $\hat{J} = \sqrt{2J+1}$ and $(L1|J)$ is a short notation for the Clebsh-Gordon coefficient $(L010|J0)$. The angular momentum coefficients are: ¹

$$\begin{aligned}
\mathcal{W}_{LSJ}(pn) &= \sqrt{2} \hat{S} \hat{J} \hat{L} \hat{l}_n \hat{j}_n \hat{j}_p (l_n L | l_p) \begin{Bmatrix} l_p & \frac{1}{2} & j_p \\ L & S & J \\ l_n & \frac{1}{2} & j_n \end{Bmatrix}, \\
\mathcal{W}_{LJ}^{(\pm)}(pn) &= \mp i (-1)^{j_p + l_n + L + \frac{1}{2}} \hat{J} \hat{L} \hat{l}_p \hat{j}_p \hat{j}_n (l_n + \frac{1}{2} \mp \frac{1}{2})^{\frac{1}{2}} (l_p L | l_n \mp 1) \\
&\times W(l_p j_p l_n j_n \frac{1}{2} J) W(L J l_n \mp 1 l_p),
\end{aligned} \tag{12}$$

and the two-body radial integrals are defined as

$$\mathcal{R}_{LL'}^{\kappa\kappa'}(pnp'n'; \omega_{J_\alpha^\pi}) = R \int dk k^{2+\kappa} v(k; \omega_{J_\alpha^\pi}) R_L^0(pn; k) R_{L'}^{\kappa'}(p'n'; k), \quad \kappa' = 0, 1, \pm, \tag{13}$$

where

$$v(k; \omega_{J_\alpha^\pi}) = \frac{2}{\pi} \frac{1}{k(k + \omega_{J_\alpha^\pi})}, \quad \omega_{J_\alpha^\pi} = E_{J_\alpha^\pi} - \frac{1}{2} (E_i + E_f), \tag{14}$$

¹We use here the angular momentum coupling $|(\frac{1}{2}, l)j\rangle$.

is the "neutrino potential", and

$$\begin{aligned} R_L^\kappa(pn; k) &\equiv R_L^\kappa(l_p, n_p, l_n, n_n; k) = \int_0^\infty u_{n_p, l_p}(r) u_{n_n, l_n}(r) j_L(kr) r^{2+\kappa} dr, \\ R_L^{(\pm)}(pn; k) &= \int_0^\infty u_{n_p, l_p}(r) \left(\frac{d}{dr} \pm \frac{2l_n + 1 \pm 1}{2r} \right) u_{n_n, l_n}(r) j_L(qr) r^2 dr. \end{aligned} \quad (15)$$

There are also two additional matrix elements, namely $M_{F\omega}$ and $M_{GT\omega}$, which are obtained from M_F and M_{GT} by the replacement

$$v(k; \omega_{J_\alpha^\pi}) \rightarrow v_\omega(k; \omega_{J_\alpha^\pi}) = \frac{2}{\pi} \frac{1}{(k + \omega_{J_\alpha^\pi})^2}. \quad (16)$$

Finally the two-body state dependent particle-hole (ph) density matrix

$$\rho^{ph}(pn p' n'; J_\alpha^\pi) = \hat{J}^{-2} \langle 0_f^+ | | (a_p^\dagger a_{\bar{n}})_{J^\pi} | | J_\alpha^\pi \rangle \langle J_\alpha^\pi | | (a_{p'}^\dagger a_{\bar{n}'})_{J^\pi} | | 0_i^+ \rangle, \quad (17)$$

contains information on the wave functions of the initial ($|0_i^+\rangle$), final ($|0_f^+\rangle$), and virtual intermediate ($|J_\alpha^\pi\rangle$) states.

In particular, within the QRPA formulation, and after solving both the BCS and the RPA equations for the intermediate $(N-1, Z+1)$ nucleus [25], the two-body density matrix becomes

$$\rho^{ph}(pn p' n'; J_\alpha^\pi) = \left[u_n v_p X_{J_\alpha^\pi}(pn) + u_p v_n Y_{J_\alpha^\pi}(pn) \right] \left[u_{p'} v_{n'} X_{J_\alpha^\pi}(p' n') + u_{n'} v_{p'} Y_{J_\alpha^\pi}(p' n') \right], \quad (18)$$

where the notation has the standard meaning [2, 25].

3 $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$ decay within the single mode model

In the SMM the virtual states in the intermediate nucleus ^{48}Sc are: $[0f_{7/2}(p)0f_{7/2}(n)]_{J^+}$, where $J^+ = 0^+ \dots 7^+$ [2]. When the harmonic oscillator radial wave functions are used and the excitation energy $\omega_{J_\alpha^\pi}$ is taken to be zero, we can go a step further in the analytical calculations.

Table 1: Radial integrals $\mathcal{R}_{LJ}^{\kappa\kappa'}(pnpn; \omega_{J_\alpha^\pi})$ for the $\beta\beta_{0\nu}$ decay in ^{48}Ca . The excitation energy $\omega_{J_\alpha^\pi}$ is taken to be zero, and harmonic oscillator radial wave functions were employed with the oscillator parameter $\nu = M\omega/\hbar$. Short notation $C = \frac{33600\sqrt{2\pi}}{R\nu\sqrt{\nu}}$ has been used.

L	J	$C\nu\mathcal{R}_{LJ}^{00}$	$C\nu\mathcal{R}_{JL}^{11}$	$C\mathcal{R}_{LJ}^{20}$	$C\mathcal{R}_{JL}^{1+}$	$C\mathcal{R}_{JL}^{1-}$
0	0	37230		12870		
1	0		18615		9805	-18615
2	0			-690		
0	2			-690		
1	2		11835		16585	-11835
2	2	4734		12870		
3	2		11835		7065	-11835
4	2			6030		
2	4			6030		
3	4		8415		10485	-8415
4	4	1870		12870		
5	4		8415		5445	-8415
6	4			8910		
4	6			8910		
5	6		6435		7425	-6435
6	6	990		12870		
7	6		6435			-6435

The results for the radial integrals $\mathcal{R}_{LL'}^{\kappa\kappa'}(pnpn; \omega_{J_\alpha^\pi})$ are listed in Table 1, and after performing the summations on L, L' and J' , as indicated in eqs. (5)-(11), we get:

$$\begin{aligned}
M_F &= M_{F'} = M_{F\omega} = R\sqrt{\frac{\nu}{2\pi}} \left(\frac{g_V}{g_A}\right)^2 \sum_{J^+} A_F(J^+) \rho^{ph}(J^+), \\
M_{GT} &= M_{GT'} = M_{GT\omega} = R\sqrt{\frac{\nu}{2\pi}} \sum_{J^+} A_{GT}(J^+) \rho^{ph}(J^+), \\
M_R &= \frac{R^2}{2M_N} \sqrt{\frac{\nu^3}{2\pi}} \frac{g_V}{g_A} \sum_{J^+} [f_W A_{1R}(J^+) + A_{2R}(J^+)] \rho^{ph}(J^+), \\
M_T &= R\sqrt{\frac{\nu}{2\pi}} \sum_{J^+} A_T(J^+) \rho^{ph}(J^+), \quad M_P = 0,
\end{aligned} \tag{19}$$

where $\rho^{ph}(J^+) \equiv \rho^{ph}(pnpn; J^+)$. The coefficients $A_X(J^+)$ are given in Table 2. It should be noted that: i) in the SMM the matrix element M_P is always null, independently of the value for the excitation energy $\omega_{J_\alpha^\pi}$, and ii) while the Fermi matrix elements arise only from even multipoles, the remaining matrix elements only come from odd multipoles.

Table 2: Coefficients $A_X(J^+)$ for the matrix elements given by eq. (19).

J^+	$980A_F(J^+)$	$63700A_{GT}(J^+)$	$245A_{1R}(J^+)$	$245A_{2R}(J^+)$	$47775A_T(J^+)$
0^+	8687	0	0	0	0
1^+	0	-746499	-738	-1218	54327
2^+	1315	0	0	0	0
3^+	0	-117247	-552	-720	36751
4^+	459	0	0	0	0
5^+	0	-72575	-870	-510	21275
6^+	175	0	0	0	0
7^+	0	-91875	-2450	0	18375

In the shell model (SM) the virtual states $|J^+\rangle$ in the ^{48}Sc nucleus and the wave function $|0_f^+\rangle$ in the final nucleus ^{48}Ti , are described, respectively, as the one-particle one-hole and two-particle two-hole excitations on the ground state $|0_i^+\rangle$ in ^{48}Ca . That is:

$$\begin{aligned}
|J^+\rangle &= |[f_{7/2}(p)f_{7/2}^{-1}(n)]J^+\rangle, \quad J^+ = 0^+, 1^+ \dots 7^+, \\
|0_f^+\rangle &= \sum_{I^+} \langle I^+ | 0_f^+ \rangle |[f_{7/2}^2(p)]I^+, [f_{7/2}^{-2}(n)]I^+; 0^+\rangle, \quad I^+ = 0^+, 2^+, 4^+, 6^+,
\end{aligned} \tag{20}$$

and the two-body density reads [4] ²

$$\rho^{ph}(J^+) = -2 \sum_I \hat{I} \langle 0_f^+ | I^+ \rangle (-)^J \begin{Bmatrix} 7/2 & 7/2 & J \\ 7/2 & 7/2 & I \end{Bmatrix}. \quad (21)$$

Table 3: Amplitudes $\langle 0_f^+ | J^+ \rangle$ of the ground state wave function in ^{48}Ti and the corresponding coefficients $\rho^{ph}(J^+)$.

J^+	$\langle 0_f^+ J^+ \rangle$	$\rho^{ph}(J^+)$
0^+	0.9433	-0.0005
1^+	—	0.0426
2^+	-0.3126	0.1159
3^+	—	0.2098
4^+	-0.1092	0.3072
5^+	—	0.3533
6^+	0.0231	0.3017
7^+	—	0.1186

To get close to the results obtained by Pantis and Vergados [19] as much as possible, we use the same wave function for the state $|0_f^+\rangle$ that was employed in their work. This wave function is listed in Table 3 together with the resulting values for the densities $\rho^{ph}(J^+)$. Note that $\rho^{ph}(0^+)$ and $\rho^{ph}(1^+)$ are relatively small because of the restoration of isospin and SU(4) symmetries, respectively. Their single-particle (Hartree-Fock) values, obtained from $\langle 0_f^+ | 0^+ \rangle = 1$, are far larger than the values shown in Table 3 ($\rho^{ph}(0^+) = -\rho^{ph}(1^+) = -0.25$). On the other hand, they are identically null when these symmetries are totally restored.

The short-range correlations (SRC) between the two nucleons are taken into account via

² It is worth remembering [2] that, in the SMM, eq. (18) reduces to

$$\rho^{ph}(J^+) = \rho_{BCS}^{ph}(J^+) \left[\frac{\omega_0 + G(J^+)}{\omega_J^+} \right],$$

where $\rho_{BCS}^{ph}(J^+) = u_p v_n u_n v_p$ is the BCS value for the two-body density, ω_0 is the unperturbed proton-neutron quasiparticle energy, ω_J^+ are the QRPA energies, and $G(J^+) \equiv G(pn\bar{p}\bar{n}; J^+)$ is the particle-particle matrix element. Thus we see that, within the SMM, the last factor in this equation plays the role of the effective charge for the $\beta\beta_{0\nu}$ decay, induced by the QRPA correlations.

the correlation function [26]

$$f_{SRC}(r) = 1 - j_0(k_c r), \quad (22)$$

where $k_c = 3.93 \text{ fm}^{-1}$ is roughly the Compton wavelength of the ω -meson. The finite nucleon size (FNS) effects are introduced in the usual way, *i.e.*, by the dipole form factors in momentum space:

$$(g_{V,A})_{FNS} = g_{V,A} \left(\frac{\Lambda^2}{\Lambda^2 + k^2} \right)^2, \quad (23)$$

with $\Lambda = 850 \text{ MeV}$. The corresponding modifications of the neutrino potentials are shown in refs. [1, 4].

Table 4: Nuclear matrix elements for the decay $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$ within the single-mode shell-model calculations. We have used $\omega_{J_\alpha} = 0$ and four different results are presented: 1) (*bare*) no correlations and no nucleon form factor, 2) (*FNS*) no correlations but with nucleon form factor, 3) (*SRC*) short range correlations but without nucleon form factor, and 4) (*FNS+SRC*) correlations and nucleon form factor.

	M_{GT}	M_F	$M_{GT\omega}$	$M_{F\omega}$	$M_{GT'}$	$M_{F'}$	M_R	M_T
<u>Present Results</u>								
bare	-1.168	0.177	-1.168	0.177	-1.168	0.177	-1.435	0.330
SRC	-1.080	0.159	-1.080	0.159	-0.657	0.073	-0.105	0.284
FNS	-0.960	0.134	-0.960	0.134	-0.644	0.066	-0.929	0.312
FNS+SRC	-0.947	0.130	-0.947	0.130	-0.574	0.052	-0.796	0.309
<u>Pantis & Vergados [19]</u>								
bare	-1.216	0.185	-1.216	0.185	-1.216	0.185	-2.178	0.344
SRC	-0.859	0.108	-0.856	0.108	-0.841	0.105	-0.115	0.346
FNS	-0.986	0.134	-0.986	0.136	-0.635	0.063	-1.344	0.322
FNS+SRC	-0.731	0.117	-0.731	0.098	-0.532	0.055	-0.324	0.330

Present results are confronted with those obtained by Pantis and Vergados [19] in Table 4. They used a somewhat different approximation for the SRC and therefore it is plausible that our matrix elements do not fully agree with theirs in the second and fourth case. In the other two cases, they should be identical, but they are not! The difference is particularly pronounced for the recoil matrix elements M_R . The reason for the discrepancies could be the values used

for the harmonic oscillator parameter $\nu = M\omega/\hbar$ and the nuclear radius R ; we have utilized $\nu = 0.916A^{-1/3} \text{ fm}^{-2}$ and $R = 1.2A^{1/3} \text{ fm}$.³

Anyhow it is worth noting that in both calculations the FNS effects and the SRC act coherently on the Fermi (F) and Gamow-Teller (GT) moments, in the sense that their combined effects always diminish them more than when they are acting individually. This, however, does not happen with M_R , in which case the FNS+SRC values turn out to be significantly larger than the SRC ones. The explanation for this somewhat curious behavior of the recoil matrix element was given by Tomoda *et al.*, [7, 27] and is as follows. The contribution of the weak magnetism in (9) can be decomposed into the central and tensor parts [7]. The central part is the dominant one, and within the closure approximation and for $\omega_{J_\alpha} = 0$, it can be rewritten in the form:⁴

$$M_{RC}(bare) = -\frac{4\pi R^2}{3M_N} \frac{f_W g_V}{g_A} \langle F | \sum_{mn} \tau_m^+ \tau_n^+ \boldsymbol{\sigma}_m \cdot \boldsymbol{\sigma}_n \delta(\mathbf{r}_m - \mathbf{r}_n) | I \rangle, \quad (24)$$

This matrix element is totally killed by the SRC (22) and therefore

$$M_{RC}(SRC) = -\frac{4\pi R^2}{3M_N} \frac{f_W g_V}{g_A} \langle F | \sum_{mn} \tau_m^+ \tau_n^+ \boldsymbol{\sigma}_m \cdot \boldsymbol{\sigma}_n \delta(\mathbf{r}_m - \mathbf{r}_n) f_{SRC}(\mathbf{r}_m - \mathbf{r}_n) | I \rangle \equiv 0. \quad (25)$$

The k^2 dependence of the form factors (23) distributes the δ -function over a finite region [7, 26], *i.e.*,

$$\delta(\mathbf{r}) \xrightarrow{FNS} \frac{\Lambda^3}{64\pi} e^{-\Lambda r} \left[1 - \Lambda r + \frac{1}{3}(\Lambda r)^2 \right]. \quad (26)$$

Consequently, the matrix element (24) decreases ($M_{RC}(FNS) \leq M_{RC}(bare)$) and $M_{RC}(FNS + SRC) \neq 0$.

³Our definitions for the nuclear matrix elements $M_F, M_{F'}, M_{F\omega}$ and M_R agree with those of Pantis and Vergados [19] only for $g_A = g_V$. As we used here $g_A = 1.254$, the results listed in their Table 1 have been renormalized accordingly.

⁴The following relation has been used:

$$\delta(\mathbf{r} - \mathbf{r}') = \frac{2}{\pi} \sum_{lm} Y_{lm}(\hat{r}) Y_{lm}^*(\hat{r}') \int k^2 dk j_l(kr) j_l(kr').$$

4 QRPA calculations

We have employed a residual δ -force $V = -4\pi(v_s P_s + v_t P_t)\delta(r)$, with different strength constants v_s and v_t for the particle-hole, particle-particle and pairing channels [2, 25, 28]. The single-particle energies, as well as the pairing parameters $v_s^{pair}(p)$ and $v_s^{pair}(n)$, have been fixed by fitting the experimental pairing gaps to a Wood-Saxon potential well.

Table 5: QRPA results for the nuclear matrix elements that include both the FNS and SRC effects. An average excitation energy $\langle\omega_{J\pi_\alpha}\rangle$ of 5.0 MeV has been used in the present evaluation.

Nucleus		M_{GT}	M_F	$M_{GT\omega}$	$M_{F\omega}$	$M_{GT'}$	$M_{F'}$	M_R	M_T	M_P
^{48}Ca	present	-0.953	0.376	-1.010	0.361	-0.022	0.203	-1.888	-0.033	0.075
	ref. [15]	-0.785	0.367	-0.830	0.343	-0.765	0.395	-1.522	0.166	-0.131
^{76}Ge	present	-2.845	0.749	-2.864	0.723	-0.837	0.371	-4.863	-0.065	-0.889
	ref. [14]	-3.014	1.173	-2.912	1.025	-1.945	1.058	-3.594	0.612	0.530
	ref. [15]	-2.929	0.111	-2.683	0.111	-3.154	0.102	-7.423	0.714	-3.360
^{82}Se	present	-2.717	0.800	-2.769	0.771	-0.603	0.398	-5.147	-0.061	-0.754
	ref. [14]	-2.847	1.071	-2.744	0.939	-1.886	0.966	-3.343	0.789	0.500
	ref. [15]	-2.212	0.018	-2.124	0.029	-2.323	0.009	-3.700	-0.175	0.108
^{100}Mo	present	-2.155	0.972	-2.363	0.935	0.354	0.493	-6.150	-0.233	1.265
	ref. [14]	-0.763	1.356	-1.330	1.218	1.145	1.161	-4.528	0.823	-1.182
	ref. [15]	-0.615	0.471	-0.420	0.436	-0.722	0.512	-0.930	0.293	2.393
^{128}Te	present	-3.417	1.019	-3.476	0.980	-0.835	0.451	-6.354	-0.136	-0.560
	ref. [14]	-3.103	1.184	-3.011	1.047	-1.999	1.054	-4.371	0.583	0.483
	ref. [15]	-2.437	0.044	-2.179	0.029	-2.673	0.054	-1.522	0.748	-3.412
^{130}Te	present	-3.225	0.978	-3.271	0.938	-0.819	0.448	-5.934	-0.118	-0.560
	ref. [14]	-2.493	0.977	-2.442	0.867	-1.526	0.860	-3.736	0.574	0.387
	ref. [15]	-2.327	0.009	-2.083	-0.002	-2.553	0.016	-5.445	0.656	-3.376

As already mentioned, the proton and neutron gap equations have been solved for the intermediate $(N - 1, Z + 1)$ nucleus as in ref. [25], and we deal only with one QRPA equation. Note that in this procedure we avoid the problem of overlapping of two sets of the same intermediate states generated from initial and final nuclei [16].

Table 6: The coefficients that appearing in eq. (3) (in units of yr^{-1}), evaluated with the matrix elements given in Table 5. We have used the kinematical factors from ref. [6].

Nucleus		C_{mm}	$C_{\lambda\lambda}$	$C_{\eta\eta}$	$C_{m\lambda}$	$C_{m\eta}$	$C_{\lambda\eta}$
^{48}Ca	present	$1.13 \cdot 10^{-13}$	$7.42 \cdot 10^{-13}$	$1.47 \cdot 10^{-8}$	$-9.94 \cdot 10^{-14}$	$2.56 \cdot 10^{-11}$	$-8.10 \cdot 10^{-13}$
	ref. [15]	$1.07 \cdot 10^{-13}$	$3.68 \cdot 10^{-13}$	$6.63 \cdot 10^{-10}$	$-4.75 \cdot 10^{-14}$	$-5.20 \cdot 10^{-12}$	$-3.43 \cdot 10^{-13}$
^{76}Ge	present	$8.27 \cdot 10^{-14}$	$1.26 \cdot 10^{-13}$	$8.20 \cdot 10^{-9}$	$-4.61 \cdot 10^{-14}$	$2.56 \cdot 10^{-11}$	$-1.57 \cdot 10^{-13}$
	ref. [14]	$1.12 \cdot 10^{-13}$	$1.36 \cdot 10^{-13}$	$4.44 \cdot 10^{-9}$	$-4.11 \cdot 10^{-14}$	$2.19 \cdot 10^{-11}$	$-4.99 \cdot 10^{-14}$
	ref. [15]	$7.33 \cdot 10^{-14}$	$1.12 \cdot 10^{-13}$	$3.22 \cdot 10^{-9}$	$-4.49 \cdot 10^{-14}$	$-1.54 \cdot 10^{-11}$	$-2.11 \cdot 10^{-13}$
^{82}Se	present	$3.48 \cdot 10^{-13}$	$1.14 \cdot 10^{-12}$	$3.65 \cdot 10^{-8}$	$-2.47 \cdot 10^{-13}$	$8.83 \cdot 10^{-11}$	$-1.39 \cdot 10^{-12}$
	ref. [14]	$4.33 \cdot 10^{-13}$	$1.01 \cdot 10^{-12}$	$1.54 \cdot 10^{-8}$	$-1.60 \cdot 10^{-13}$	$6.37 \cdot 10^{-11}$	$-3.84 \cdot 10^{-13}$
	ref. [15]	$1.75 \cdot 10^{-13}$	$4.78 \cdot 10^{-13}$	$1.53 \cdot 10^{-9}$	$-8.77 \cdot 10^{-14}$	$-1.31 \cdot 10^{-11}$	$-9.32 \cdot 10^{-13}$
^{100}Mo	present	$4.47 \cdot 10^{-13}$	$1.93 \cdot 10^{-12}$	$6.58 \cdot 10^{-8}$	$-4.22 \cdot 10^{-13}$	$1.32 \cdot 10^{-10}$	$-2.03 \cdot 10^{-12}$
	ref. [14]	$2.05 \cdot 10^{-13}$	$1.05 \cdot 10^{-12}$	$3.50 \cdot 10^{-8}$	$-1.61 \cdot 10^{-13}$	$6.48 \cdot 10^{-11}$	$7.03 \cdot 10^{-13}$
	ref. [15]	$6.77 \cdot 10^{-14}$	$3.28 \cdot 10^{-14}$	$2.91 \cdot 10^{-9}$	$-5.64 \cdot 10^{-15}$	$-1.11 \cdot 10^{-11}$	$2.45 \cdot 10^{-14}$
^{128}Te	present	$3.60 \cdot 10^{-14}$	$1.12 \cdot 10^{-14}$	$3.14 \cdot 10^{-9}$	$-1.10 \cdot 10^{-14}$	$1.42 \cdot 10^{-11}$	$-1.25 \cdot 10^{-14}$
	ref. [14]	$3.36 \cdot 10^{-14}$	$7.39 \cdot 10^{-15}$	$1.50 \cdot 10^{-9}$	$-4.86 \cdot 10^{-15}$	$9.46 \cdot 10^{-12}$	$-1.87 \cdot 10^{-15}$
	ref. [15]	$1.36 \cdot 10^{-14}$	$4.32 \cdot 10^{-15}$	$8.17 \cdot 10^{-10}$	$-5.24 \cdot 10^{-15}$	$-4.73 \cdot 10^{-12}$	$-8.51 \cdot 10^{-15}$
^{130}Te	present	$7.83 \cdot 10^{-13}$	$1.97 \cdot 10^{-12}$	$5.66 \cdot 10^{-8}$	$-5.19 \cdot 10^{-13}$	$1.75 \cdot 10^{-10}$	$-2.34 \cdot 10^{-12}$
	ref. [14]	$5.34 \cdot 10^{-13}$	$1.05 \cdot 10^{-12}$	$2.25 \cdot 10^{-8}$	$-2.17 \cdot 10^{-13}$	$9.10 \cdot 10^{-11}$	$-4.13 \cdot 10^{-13}$
	ref. [15]	$3.02 \cdot 10^{-13}$	$7.44 \cdot 10^{-13}$	$1.61 \cdot 10^{-8}$	$-2.28 \cdot 10^{-13}$	$-6.24 \cdot 10^{-11}$	$-1.49 \cdot 10^{-12}$

The nuclei ^{76}Ge , ^{82}Se , ^{100}Mo , ^{128}Te and ^{130}Te have been evaluated within an eleven dimensional model space including all single particle orbitals of oscillator shells $3\hbar\omega$ and $4\hbar\omega$

plus the $0h_{9/2}$ and $0h_{11/2}$ orbitals from the $5\hbar\omega$ oscillator shell. In the case of ^{48}Ca we work in a seven dimensional model space including all the orbitals in the major shells $2\hbar\omega$ and $3\hbar\omega$. Here, the experimental single-particle energies have been used for the orbitals $1p_{1/2}$, $0f_{5/2}$, $1p_{3/2}$, $0f_{7/2}$, $1s_{1/2}$ and $0d_{3/2}$, while for the remaining orbitals a single-particle energy spacing of $\hbar\omega = 41 \text{ A}^{-1/3} \text{ MeV}$ has been assumed. Finally, both the $T = 1$ and $T = 0$ proton-neutron interaction strengths in the particle-particle channel have been set by following the recipe introduced in ref. [28].

Our results for nuclear matrix elements are compared in Table 5 with those obtained by MBK and PSVF. In both works configuration spaces similar to ours were employed, and the FNS effect was included in the way we have done it (see eq. (23)). Yet there are two differences that could in principle be important: i) instead of the δ -force, they have used the G-matrix (derived from the nucleon-nucleon potential) as the residual interaction, and ii) their correlation function is not that given by eq. (22). In spite of these dissimilarities, our results concord surprisingly well with those obtained by MBK, except for M_T and M_P .⁵ The major difference is found in ^{100}Mo , but we know that this is a "difficult" nucleus from the nuclear structure point of view, because of the collapse of the QRPA in the physical region of the particle-particle $T = 0$ strength. Moreover this effect is amplified by the SRC. The agreement with the PSVF calculation is only good in the case of the Gamow-Teller moments. Note that the last can also be say for the concordance between the MBK and PSVF results.

The coefficients C_{ij} , defined in eq. (3) and evaluated with the matrix elements given in Table 5, are compared in Table 6. Kinematical factors from ref. [6] have been used in our calculations. Obviously, all the above mentioned differences between the matrix elements are reflected on the calculated C_{ij} values. However, the spread between the entries in the same row in Table 6 is smaller than the spread of the values in Table 5. This is because the effect of the matrix elements M_T and M_P is comparatively small.

Finally, Table 7 gives the constraints on the Majorana neutrino mass and the right-handed coupling constants, deduced from the most recent experimental bounds for the $\beta\beta_{0\nu}$ half-lives, and the present evaluation of the nuclear matrix elements. It should be kept in mind that in doing so we have used the bare value $g_A = 1.254$ for the axial-vector coupling constant, and

⁵Except the tensor moment M_T , the MBK matrix elements agree remarkably well with those obtained by Tomoda and Faessler [13]. These authors have not evaluated M_T , since a negligible small value for it was obtained previously [27] in a projected mean-field approach.

that the upper limits for the lepton violating terms shown in Table 7 do not simply scale as g_A^4 .

Table 7: Experimental half-lives for the neutrinoless double beta decay and upper limits on the Majorana neutrino mass $\langle m_\nu \rangle$, and the right-handed current coupling strengths $\langle \lambda \rangle$ and $\langle \eta \rangle$.

Nucleus	$T_{0\nu}(exp)$ [yr]	$ \langle m_\nu \rangle $ [eV]	$ \langle \lambda \rangle $	$ \langle \eta \rangle $
^{48}Ca	$> 1.1 \cdot 10^{22}$ ^{a)}	< 15	$< 1.1 \cdot 10^{-5}$	$< 7.9 \cdot 10^{-8}$
^{76}Ge	$> 1.2 \cdot 10^{25}$ ^{b)}	< 0.51	$< 8.1 \cdot 10^{-7}$	$< 3.2 \cdot 10^{-9}$
^{82}Se	$> 2.7 \cdot 10^{22}$ ^{a,b)}	< 5.3	$< 5.7 \cdot 10^{-6}$	$< 3.2 \cdot 10^{-8}$
^{100}Mo	$> 5.2 \cdot 10^{22}$ ^{a,b,c)}	< 3.4	$< 3.2 \cdot 10^{-6}$	$< 1.7 \cdot 10^{-8}$
^{128}Te	$> 7.7 \cdot 10^{24}$ ^{d)}	< 0.97	$< 3.4 \cdot 10^{-6}$	$< 6.4 \cdot 10^{-9}$
^{130}Te	$> 8.2 \cdot 10^{21}$ ^{e)}	< 6.4	$< 7.9 \cdot 10^{-6}$	$< 4.6 \cdot 10^{-8}$

^{a)} (laboratory data) ref. [20]

^{b)} (laboratory data) ref. [21]

^{c)} (laboratory data) ref. [22]

^{d)} (geochemical data) ref. [23]

^{e)} (laboratory data) ref. [24]

5 Concluding remarks

Nuclear moments for the neutrinoless double beta decay have been evaluated numerically for several nuclei, using the formalism that we have recently developed. Simple analytic expressions for the $\beta\beta$ decay of ^{48}Ca , that follow from this formalism in the single mode model, are also presented. The results shown in Table 2 are useful, not only for testing the full numerical calculations, but also for checking the consistency with other formalisms [5, 6, 8, 9]. In fact, it would be highly desirable to find out whether these formalisms lead to numbers shown in Table 2. This would be a simple and definite test for all nuclear matrix element except for the M_P . However, even in this case, a simple model can be framed for confronting different formalisms with each other.

The present work differs from similar QRPA studies, not only in the $\beta\beta_{0\nu}$ formalism, but also in the residual interaction. Namely, we have used a simple δ -force, instead of the G-matrix that is currently employed. The fact that our results are equivalent to those obtained by MBK and PSVF clearly shows that the $\beta\beta_{0\nu}$ half-lives are not very sensitive to details of the nuclear force. In other words, the so called "realistic interactions" are in no way the panacea for the nuclear structure evaluation of the $\beta\beta$ processes, as it has been proclaimed by some authors for a long time. The reason for that is the crucial role played by the restoration of the isospin and SU(4) symmetries, produced by the residual interaction, in tailoring the Fermi and Gamow-Teller transitions strengths, respectively [2]. This restoration mechanism is not limited to the RPA-like models [2, 29, 30, 31], but also occurs in the shell model calculation. (One simple example has been discussed in Sec. 3.) We are convinced that the nuclear structure issue, involved in $\beta\beta$ -decays will still keep us busy for a long time, and that definitively it cannot be solved by a simple minded employment of "good" residual interactions. Neither the renormalized nor self consistent RPA methods are able to fix this problem [32].

An alternative technique for examining the $\beta\beta$ transitions could be the relativistic RPA, which has recently been successfully applied to the description of the isobaric analogue and Gamow-Teller resonances in closed shell nuclei [33]. We are planning to analyze the consequences of such an approach.

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